DYNAMICS OF THE BEHAVIOR OF NEMATIC FILMS IN GAS-DYNAMIC FLOWS

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The application of liquid crystals in aerodynamic studies helps to expand the inventory of flow diagnostic methods [1-3], owing to the unique ability of liquid crystals to change their optical properties under the influence of extremely small external disturbances. A thin liquid crystal film deposited on the surface of a model in aerodynamic experimental work is capable of altering its properties under the influence of temperature and mechanical shear. Whereas the tendency of liquid crystals to change color with variations in the temperature has been investigated and made the transition to practical application, the influence of shear on the optical properties of liquid crystals has not received adequate attention.

In the present article we attempt to broaden research in this direction. We have analyzed the variations of the optical properties of nematic liquid crystals under the influence of flow induced by shear movement of a thin film of a crystal having a specific initial molecular orientation.

An ultimately simple order is inherent in all types of nematic liquid crystals: their molecules tend to line up parallel with a common axis, which is characterized by a single vector **n** known as the director. When a nematic liquid crystal (NLC) film is exposed to a stream of air, a shear flow is created in the interior of the film (Fig. 1), changing the orientation of the director. The requisite equations describing the reorientation of the director of a NLC in the one-dimensional case are obtained from the Leslie–Ericksen equations and have the form [4]

$$
\begin{gathered}
2f(\theta)\frac{d^2\theta}{dz^2}+\frac{df}{d\theta}\left(\frac{d\theta}{dz}\right)^2+(\lambda_1+\lambda_2\cos 2\theta)\frac{dv}{dz}=0 \quad (0\leqslant z\leqslant L),\\
dv/dz=b/g(\theta),\ f(\theta)=k_1\sin^2\theta+k_3\cos^2\theta,\\
g(\theta)=(1/2)[(\mu_1/2)\sin^2 2\theta+(\mu_5-\mu_2)\sin^2\theta+(\mu_6+\mu_3)\cos^2\theta+\mu_4].
\end{gathered} \tag{1}
$$

The system (1) describes the steady laminar flow of a NLC with velocity v in the x direction. Here $v_x = v(z)$, $v_y = 0$, $v_z = 0$, $\theta = \theta(z)$ is the angle of deviation of the director from the initial (unperturbed) direction relative to the x axis (see Fig. 1), b is the shear stress constant, μ_i denotes the Leslie viscosity constants, $\lambda_1 = \mu_2 - \mu_3$, $\lambda_2 = \mu_5 - \mu_6$, k_i are the Frank elastic constants, and L is the thickness of the NLC film. The orientation of the director is specified by a single vector with components $n_x = \cos\theta(z)$, $n_y = 0$, $n_z = \sin\theta(z)$.

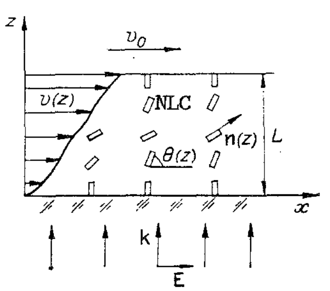

Fig. 1

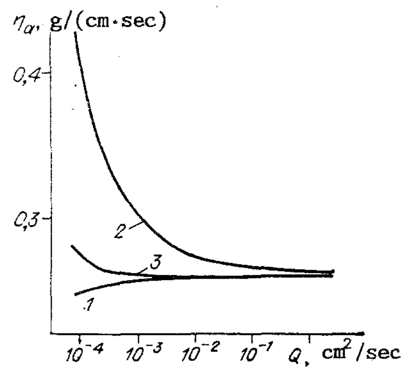

Fig. 2

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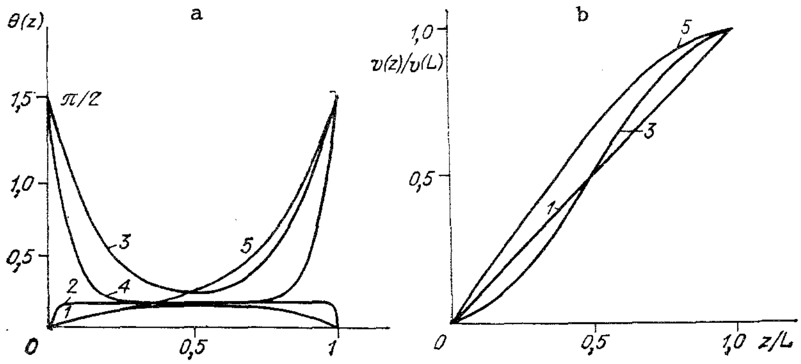

Fig. 3

The boundary conditions for the angle θ are varied, and the velocity at the boundaries is specified in the form

$$v|_{z=0} = 0, \; v|_{z=L} = v_0.$$

For the ensuing discussion it is useful to define the apparent viscosity η_a in terms of the flow rate Q according to the formula

$$\eta_a = bL^2/2Q, \tag{2}$$

where $Q = \int_0^L v(z)\,dz$. The velocity v_0 of the upper boundary in this case is related to the shear stress constant b by the equation

$$v_0 = b\int_0^L \frac{dz}{g(\theta)}. \tag{3}$$

The quantity b is varied in the calculations. Equations (1) are solved numerically for the following values of the constants: $k_1 = 7.1 \cdot 10^{-7}$ dyne; $k_3 = 9.2 \cdot 10^{-7}$ dyne; $\mu_i = 0.065$, -0.775, -0.012, 0.832, 0.463, -0.344 g/cm·sec; L = 100 μm.

Figure 2 shows the variation of η_a as a function of Q for various molecular orientations of the liquid crystal in the bounding planes, where the curves are numbered as follows: 1) $\theta(0) = 0$, $\theta(L) = 0$; 2) π/2, π/2; 3) 0, π/2. A strong dependence of η_a on the boundary conditions is observed, particularly for normal (homeotropic) orientation of the NLC molecules at the boundaries (curve 2). This conclusion is further supported by Fig. 3a, which shows the profile of the angle θ(z): 1) $Q = 8.6 \cdot 10^{-5}$ 2) 2.72; 3) $8.4 \cdot 10^{-5}$; 4) 2.68; 5) $7.65 \cdot 10^{-5}$ cm²/sec. The plateaus of curves 2 and 4 correspond to the asymptotic angle $\theta = \theta_0$ determined from the relation [4] $\tan^2\theta_0 = \mu_3/\mu_2$, which is valid at high flow velocities (saturation condition).

Figure 3b illustrates the dependence of the NLC flow velocity on the coordinate z. A strong departure from linearity is evident. Note that curve 3 has an inflection point in the center of the region occupied by the crystal. Such profiles are unstable in the case of orindary Newtonian fluids [5]. In our situation involving the shear flow of a NLC, the calculated profiles are found to be stable. In our opinion, this fact can be counted as one of the phenomena associated with the nonlinearity (even for laminar flows) of the hydrodynamic laws of non-Newtonian fluids.

The stability of the flow is established on the basis of the solution of the dynamical problem as in [6]. Its verification essentially entails varying the initial conditions and then calculating the differences in the steady-state solutions. Stability is inferred from the fact that this difference tends to zero with time. The dynamical equations are derived from the same Leslie–Ericksen equations [4] as in the derivation of Eqs. (1), but without the assumption that the flow is steady.

We now consider the variations that take place in a diagnostic wave of the extraordinary (e) type transmitted through a flow-reoriented NLC film. The phase shift ψ in normal inci-

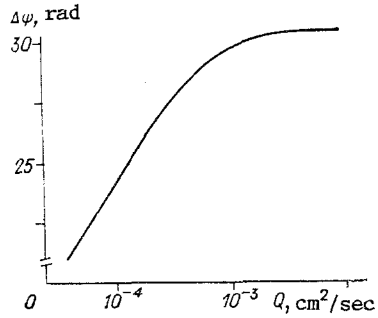

Fig. 4

dence of the diagnostic e-wave is expressed by the following integral in the given geometry:

$$\psi = k_0 \int_0^L n_e(z)\,dz,$$

where $k_0 = 2\pi/\lambda$ (λ is the light wavelength in vacuum), and n_e is the e-wave refractive index, which depends on the orientation of the director and has the form

$$n_e = \frac{n_\perp n_\parallel}{\sqrt{n_\parallel^2 \sin^2\theta(z) + n_\perp^2 \cos^2\theta(z)}}$$

($n_\parallel$ and $n_\perp$ are the refractive constants). The change of phase in reorientation of the director from an initial homeotropic orientation is given by the relation

$$\Delta\psi = k_0\left(n_\perp L - \int_0^L n_e(z)\,dz\right).$$

Figure 4 shows the calculated dependence of $\Delta\psi$ on Q for $\lambda = 0.5$ μm, $n_\parallel = 1.75$, and $n_\perp = 1.54$. It is evident that very slight changes in the flow rate are accompanied by an appreciable variation of the phase shift. The data obtained here lead to the assumption that the phase shift in a diagnostic e-wave can be used to determine, for example, surface friction. The following measurement scheme is proposed: an intermediate (auxiliary) flow parameter Q is estimated from experimental data on the behavior of the phase shift $\Delta\psi$ (Fig. 4). The shear stress constant b is calculated from the dependence of the apparent viscosity η_a on Q according to Eq. (2). This measurement scheme relies on knowledge of the thickness L on the NLC film, which changes under the influence of flow and might be unknown. The difficulty can be surmounted by recording the phase shift simultaneously at two different points of the film and then calculating the two parameters L and b. This technique has been used previously (see, e.g., [7]). Of course, the proposed procedure is in need of experimental verification and correction.

LITERATURE CITED

1. E. J. Klein, "Application of liquid crystals to boundary layer flow visualization," AIAA Paper No. 68-3767, AIAA, New York (1968).
2. G. M. Zharkova, "The application of liquid crystals in experimental acrodynamics," Fluid Mech. Sov. Res., 9, No. 3 (1980).
3. P. D. Call and B. J. Holmes, "Liquid crystals for high altitude inflight boundary layer flow visualizations," AIAA Paper No. 86-2592, AIAA, New York (1986).
4. S. Chandrasekhar, Liquid Crystals, Cambridge Univ. Press, New York (1977); paperback (1980).
5. C. C. Lin, The Theory of Hydrodynamic Stability, 2nd ed., Cambridge Univ. Press, Oxford (1966).
6. N. G. Preobrazhenskii and S. I. Trashkeev, "Multimode oscillations of the director of a nematic liquid crystal in the light field of an obliquely incident ordinary wave," Opt. Spektrosk., 62, No. 6 (1987).
7. L. H. Tanner and L. G. Blows, "A study of the motion of oil film on surfaces in air flow with application to the measurement of skin friction," J. Phys. E, 9 (1976).